The Electron and the Holographic Mass Solution*

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Abstract

A computation of the electron mass is found utilizing a generalized holographic mass solution in terms of quantum electromagnetic vacuum fluctuations. The solution gives a clear insight into the structure of the hydrogen Bohr atom, in terms of the electron cloud and its relationship to the proton and the Planck scale vacuum fluctuations. Our electron mass derivation is accurate to within 0.000000002 x 10^{-28} g (99.99999998%) of the CODATA value. As a result, an elucidation of the source of the fine structure constant α, the Rydberg constant R_∞, and the proton-to-electron mass ratio μ is determined to be in terms of vacuum energy interacting at the Planck scale.

Introduction

Measurements of the electron mass are typically determined utilizing penning traps, where the latest CODATA value is given as $9.10938356(11)\times 10^{-28} g$ [1]. However, although measurements are extremely precise a satisfactory derivation from first principles has yet to be found and thus the nature of the electron remains a mystery.

The Bohr model considers the electron as an extended source while relativistic quantum field theory (QED) treats the electron and positron as point particles with no internal structure yet each possessing an intrinsic angular momentum, or spin. The point-like nature of the electron, in quantum field theories, leads to an infinite bare mass and bare charge. To agree with measurement, the mass of the electron is subsequently given in terms of two infinities, the bare mass and the radiative corrections. The standard mass of the electron is therefore generally calculated from the definition of the Rydberg constant for an atom with nucleus of infinite mass,

$$m_e = \frac{2R_\infty h}{c\alpha^2} = 9.10938358396287\times 10^{-28} g \qquad \text{Eqn. 1}$$

This value agrees with the CODATA value to within 0.00000002 x 10^{-28} g, which is a less precise agreement than our holographic derivation by an order of magnitude. More importantly this standard derivation, in terms of the Rydberg constant, does not reveal the nature or structure of the electron, or give us insight into the source of mass and charge.

There is currently no evidence to support the point-like view, although Crater and Wong suggest that such a view would be supported if the existence of the peculiar ground singlet state ${}^1S_0$ is found [2] or at the least could provide an experimental limit on the point-like nature. As noted by Wilczek, the complexities and structure only emerge above the energy threshold of ~ 1MeV, below which it 'appears' point-like and structure-less [3].

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In any case, although the position and momentum can only be defined in terms of a probability cloud, the quantum behavior of the electron is successfully calculated by the current standard model. The most precise prediction being that of the g-factor [4] [5] where the observed deviation [6], known as the anomalous magnetic moment, was subsequently attributed to quantum fluctuations [7]. Quantum corrections are also expected for an electric field – but as yet no such field has been detected. Based on charge-parity (CP) violating components the standard model assumes an upper limit on the electron electric dipole moment (EDM) of $d_e \leq 10^{-38} q$ cm, [8] which is smaller than current experimental sensitivities. However recent experiments confirm a non-zero EDM e.g. [9] and [10] who find $d_e < 10.5 \times 10^{-28} q$ cm and $d_e < 6.05 \times 10^{-25} q$ cm, respectively, suggesting the standard model is incomplete and there must be other sources of CP violation. Higher EDMs are predicted by extensions to the standard model e.g. supersymmetric models, which predict $d_e > 10^{-26} q$ cm [11].

Successful predictions allow us to confirm and improve upon existing models and thus gain greater insight into the structure and characteristics of the electron. Defining these terms from fundamental principles is therefore of great importance as not only will it provide information about the structure of subatomic particles but also the source of mass itself. The current source of mass, according to the standard model, is through the interaction with the Higgs field, where as the 'mass terms' violate gauge symmetry, a measurable mass is only acquired through symmetry breaking.

In earlier work [12], utilizing a generalised holographic solution, the mass of the proton was successfully computed. As a result, a precise charge radius value was found which is within an 1σ agreement with the latest muonic measurements of the charge radius of the proton [13], relative to a 7σ variance in the standard approach [14]. We now extend our holographic solution in an attempt to deepen our understanding of the electron and its relationship with the Planck scale vacuum fluctuations. Specifically, our result computes the mass of the electron in terms of surface-to-volume ratios of Planck oscillator information bits with an accuracy of within 0.000000002 x 10^{-28} g of the CODATA value.

This new definition, with a source for mass, successfully predicts the energy levels for the currently known quantum states of the Hydrogen atom, as well as the atomic number for the n=1 state of all known atoms.

The Holographic Principle and the proton mass

The Bekenstein conjecture, first suggested by Jacob Bekenstein in the early 1970's, proposed that the entropy S or information contained in a given region of space, such as a black hole, is proportional to its surface horizon area [15] [16] [17]. Based on the laws of thermodynamics and the prediction of Hawking radiation, Hawking inferred and subsequently set the constant of proportionality to be ¼ of the surface horizon [18]. The Bekenstein-Hawking entropy of a black hole expressed in units of Planck area is thus given as,

$$S = \frac{A}{4\ell^2} \qquad \text{Eqn. 2}$$

where the Planck area, ℓ^2 is taken as one unit of entropy.

Bekenstein [19] further argued for the existence of a universal upper bound for the entropy of an arbitrary system with maximal radius r,

$$S \leq \frac{2\pi r E}{\hbar c} \qquad \text{Eqn. 3}$$

and found that this maximal bound is equivalent to the Bekenstein-Hawking entropy for a black hole (assuming $E = mc^2$). This confirmed the long suspected assumption that black holes have the maximum entropy for a given mass and size, which along with unitarity arguments led to the holographic principle of ‘t Hooft, where one bit of information is encoded by one Planck area [20] [21].

Following the holographic principle of ‘t hooft [21], based on the Bekenstein-Hawking formulae for the entropy of a black hole [22] [23], Haramein [12] [13] defines the holographic bit of information as the oscillating Planck spherical unit (PSU), given as

$$PSU = \frac{4}{3}\pi r_\ell^3 \qquad \text{Eqn. 4}$$

where $r_\ell = \frac{\ell}{2}$ and ℓ is the Planck length.

These PSUs, or Planck voxels, tile along the area of a spherical surface horizon, producing a holographic relationship with the interior information mass-energy density.

Theories of quantum gravity suggest that the quantum entropy of a black hole may not exactly equal to A/4 [24]. Indeed, in Haramein’s generalized holographic approach, he suggests that the entropy of a spherical surface horizon should be calculated in spherical bits and thus defines the surface entropy in terms of PSUs, such that,

$$S = \frac{A}{\pi r_\ell^2} \qquad \text{Eqn. 5}$$

where the Planck area, taken as one unit of entropy, is the equatorial disk of a Planck spherical unit, πr_ℓ^2. We note that in this definition, the entropy is slightly greater (~ 2.5 times) than that set by the Bekenstein Bound, and the proportionality constant is taken to be unity. To differentiate, the information/entropy encoded on the surface boundary in Haramein’s model is termed, η.

As first proposed by ‘t Hooft the holographic principle states that the description of a volume of space can be encoded on its surface boundary, with one discrete degree of freedom per Planck area, which can be described as Boolean variables evolving with time [20].

Following his definition for surface entropy, Haramein similarly defines the information/entropy within a volume of space as,

$$R = S_V = \frac{V}{\frac{4}{3}\pi r_\ell^3} = \frac{r^3}{r_\ell^3} \quad \text{Eqn. 6}$$

and finds that $R \neq \eta$, and instead $R > \eta$ for $r > 4r_\ell = 2\ell \equiv r_{\ell S}$ where $r_{\ell S}$ is the Schwarzschild radius for a black hole with a mass, $m = m_\ell$. This inequality suggests that the volume information/entropy may be non-local due to wormhole interactions as those proposed by the ER=EPR conjecture [25]. As well, it is interesting to note that $R = \eta$ only when the radius of the spherical volume is equal to the Schwarzschild radius of a black hole with a mass, $m = m_\ell$, supporting the conjecture that the Planck entity is the fundamental granular structure of space time.

Utilizing this holographic ratio given as,

$$\phi = \frac{\eta}{R} \quad \text{Eqn. 7}$$

where η is the number of PSU on the spherical surface horizon (surface entropy) and R is the number of PSU within the spherical volume (volume entropy), Haramein finds that for any black hole of radius r_S the mass can be given as,

$$m_{BH} = \frac{m_\ell}{\phi} \quad \text{Eqn. 8}$$

where m_ℓ is the Planck mass and m_{BH} is the Schwarzschild mass of a black hole, thus yielding an analogue to the Schwarzschild solution in terms of a discrete structure of spacetime at the Planck scale.

As well, when this approach is applied to the nucleonic scale a precise value for the charge radius of the proton is found within 1σ agreement with the latest muonic measurements [12] [13]

$$m_p = 2\phi m_\ell \quad \text{Eqn. 9}$$

where m_p is the proton mass.

Determining the mass of the electron

This same geometric approach is applied to the electron cloud of a hydrogen Bohr atom, where we find the mass of the electron in terms of ϕ_e,

$$m_e = \frac{1}{2\alpha}\phi_e m_\ell = 9.10938627256507 \times 10^{-28}\, g \quad \text{Eqn. 10}$$

where ϕ_e is the holographic surface-to-volume ratio in terms of PSU at the hydrogen Bohr atom radius, given as,

$$\phi_e = \frac{\eta_e}{R_e} \qquad \text{Eqn. 11}$$

where

$$\eta_e = \frac{4\pi a_0^2}{\pi r_\ell^2} \qquad \text{Eqn. 12}$$

$$R_e = \frac{a_0^3}{r_\ell^3} \qquad \text{Eqn. 13}$$

and a_0 is the Bohr radius and α is the fine structure constant.

The accuracy of the solution is within $0.00000217 \times 10^{-28} g$ (99.99997%) of CODATA 2014 [1]. However, the accuracy of our solution is restricted by the value of the Planck length which is dependent on experimental values given for the Gravitational constant, G. By calculating G in terms of fundamental units, given as,

$$G = \frac{q_\ell^2}{m_\ell^2} = \frac{\hbar c}{m_\ell^2} = 6.6740821 \times 10^{-8} g^{-1} cm^3 s^{-2} \qquad \text{Eqn. 14}$$

we find that $\ell = \sqrt{\frac{\hbar G}{c^3}} = 1.6162285183572 \times 10^{-33} cm$ Eqn. 15

Therefore, the mass of the electron in terms of ϕ_e, is given as,

$$m_e = \frac{1}{2\alpha} \phi_e m_\ell = 9.109383557931 \times 10^{-28} g \qquad \text{Eqn. 16}$$

which is now accurate to within $0.000000002 \times 10^{-28} g$ (99.99999998%) of CODATA [1].

The mass of the electron, defined in terms of the holographic surface-to-volume ratio, can be extended to include the elementary and Planck charge and orbital velocity,

$$m_e = \frac{1}{2\alpha} \phi_e m_\ell = \frac{1}{2} \frac{q_\ell^2}{q^2} \phi_e m_\ell = \frac{1}{2} \frac{v_\ell}{v_e} \phi_e m_\ell \qquad \text{Eqn. 17}$$

where

$$\alpha = \frac{q^2}{\hbar c} = \frac{q^2}{q_\ell^2} = \frac{\alpha\hbar c}{\hbar c} = \frac{v_e}{v_\ell}$$ Eqn. 18

When we further extend this solution for the n=1 state we find that, at radii of successively smaller fractions of a_0, the holographic mass solution yields values of $2m_e$, $3m_e$ … Nm_e, where $r = \frac{a_0}{N}$ (see Figure 1).

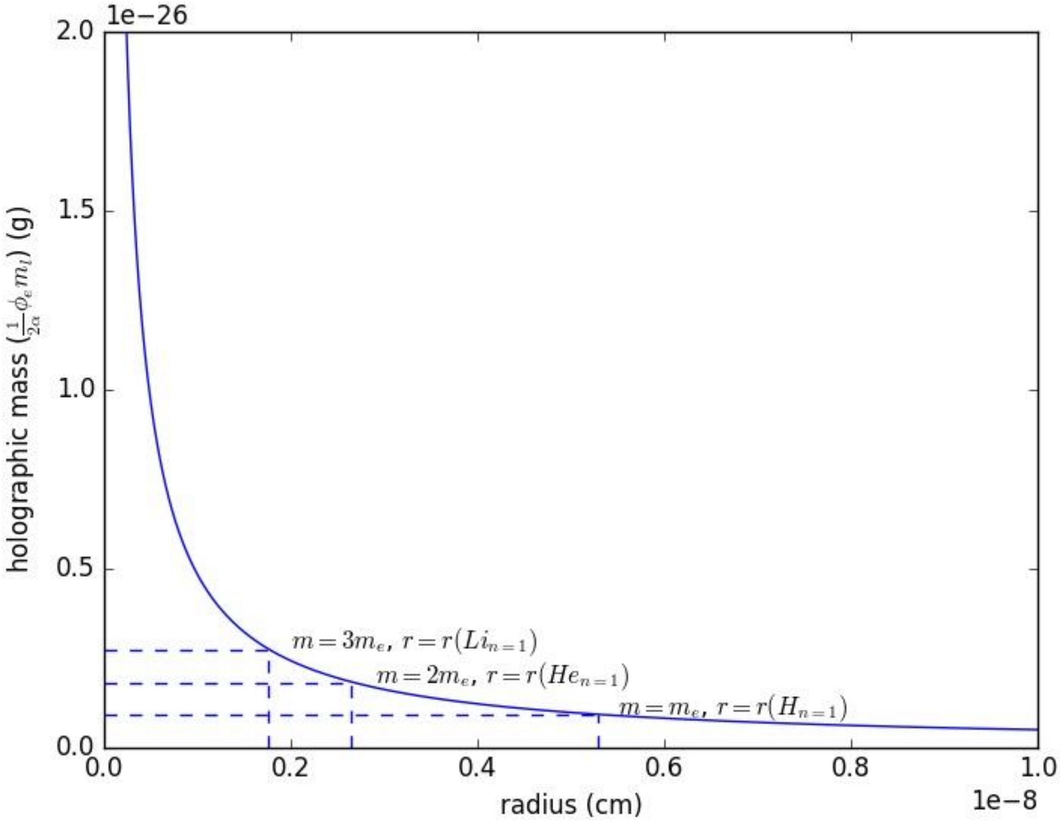

Figure 1: Graph to show the holographic mass solution as a function of radius. Note: the holographic mass is equal to *Nm*$_e$ at corresponding radii of *a*$_0$/*N*. For example, the holographic mass: equals the mass of one electron at a radius of the hydrogen atom in its n=1 state; equals the mass of two electrons at a radius of the helium atom in its n=1 state; equals the mass of three electrons at a radius of the Lithium atom in its n=1 state, and so on. Note this relationship is only shown on the graph for the first three elements, but continues for all known elements.

We can thus put Eqn. 10 in the form

$$Nm_e = \frac{1}{2\alpha}\phi_e m_\ell$$ Eqn. 19

and recognize N as being the atomic number Z, such that

$$Zm_e = \frac{1}{2\alpha}\phi_e m_\ell \qquad \text{Eqn. 20}$$

In terms of velocity or charge we thus have,

$$m_e = \frac{1}{2Z\alpha}\phi_e m_\ell = \frac{1}{2}\frac{q_\ell^2}{Zq^2}\phi_e m_\ell = \frac{1}{2}\frac{v_\ell}{v_e}\phi_e m_\ell \qquad \text{Eqn. 21}$$

where

$$v_e = Z\alpha c \qquad \text{Eqn. 22}$$

From this holographic mass solution, we are thus able to calculate the total mass of the electrons for each element, without the need for adding the atomic mass number, Z. We instead find that the atomic mass number Z is a natural consequence of the holographic solution. As a result, a picture develops in which the structure of the Bohr atom and the charge and mass of both the proton and the electron are consequences of spin dynamics in the co-moving behaviour of the Planck scale granular structure of spacetime. This suggests that the confinement for the electron is a result of the quantum gravitational force exerted by the dynamics of the vacuum at the Planck scale. The electrostatic force can thus be accounted for in the same way the strong force is accounted for in the case of the proton [12] [13], where in both cases, the proton and the electron confinement is the result of the quantum gravitational force exerted by the granular Planck scale structure of spacetime.

The current quantum understanding resolves the hierarchy bare mass problem for the electron mass through the consideration of antimatter where positron and electron pairs pop in and out of the vacuum. These virtual particles smear out the charge over a greater radius such that the bare mass energy is cancelled by the electrostatic potential, where the greater the radius the lesser the need for fine tuning. In the solution presented here the electron is extended to a maximal radius of a_0 and we are able to demonstrate that the mass of the electron is a function of the Planck vacuum oscillators surface to volume holographic relationship, over this region of space time. The hierarchy bare mass problem is thus resolved by considering Planck vacuum oscillators acting coherently extending over a region of space equivalent to the Bohr hydrogen atom.

In much the same way the electron analogy is proposed to resolve the Higgs hierarchy problem, with the inclusion of virtual supersymmetric particles, we could also assume that the surface to volume holographic relationship in the Higgs region of space would solve for the mass of the Higgs, where the Higgs radius would be of the order, $r_\ell < r_{Higgs} < r_p$.

The hierarchy problem associated with the mass of the electron and the mass of the proton can also be understood in terms of the surface to volume holographic ratio over their respective commoving regions of space, where the greater the radius the smaller the mass. The mass is thus a direct function of the commoving behaviour of the Planck vacuum, where the spin and mass decrease as a function of radius.

Deriving the Rydberg constant, the fine structure constant and the proton to electron mass ratio

From this geometric solution for the electron, we can derive the Rydberg constant, R_∞, the fine structure constant, α and the proton to electron mass ratio, μ, in terms of ϕ_e.

The standard formula for the mass of the electron is,

$$m_e = \frac{2R_\infty h}{c\alpha^2} = 9.10938358396287 \times 10^{-28} g \qquad \text{Eqn. 1}$$

which can be reduced to,

$$m_e = \frac{2R_\infty h}{c\alpha^2} = \frac{4\pi\ell m_\ell R_\infty}{\alpha^2} \qquad \text{Eqn. 23}$$

Equating this (Eqn. 23) with the geometric solution (Eqn. 10) gives,

$$m_e = \frac{4\pi\ell m_\ell R_\infty}{\alpha^2} = \frac{1}{2\alpha}\phi_e m_\ell \qquad \text{Eqn. 24}$$

and thus

$$R_\infty = \frac{\alpha\phi_e}{8\pi\ell} = 1.09737315686404 \times 10^5 cm^{-1} \qquad \text{Eqn. 25}$$

This definition offers a geometric solution for the Rydberg constant with an improved accuracy within 0.0000236 cm^{-1} (99.999999978%) of the experimentally determined CODATA value. It should be noted, that this value is 10% more accurate than that found from the standard definition, which is accurate to within 0.0000263 cm^{-1} (99.999999976%). The Rydberg constant is considered to be one of the most well-determined physical constants, with an accuracy of 7 parts to 10^{12} and is thus used to constrain the other physical constants [26]. However, as the same spectroscopic experiments are used to determine both the charge radius of the proton and the Rydberg constant, the recent muonic measurements of the charge radius of the proton implies that the Rydberg constant would change by $4-5\sigma$ [27] [28] [29]. This is known as the proton radius puzzle. The standard formula for the Rydberg constant is given as,

$$R_\infty = \frac{m_e\alpha^2 c}{2h} \qquad \text{Eqn. 26}$$

so any change in the experimentally determined value for the Rydberg constant could thus have an effect on m_e, c and h. However, in all cases the accuracy of m_e, c and h are known to a lesser degree of accuracy and would thus not be effected by a shift of $4-5\sigma$ in the Rydberg constant. This new definition, as shown in Eqn. 25, gives us a physical description and insight into how R_∞ emerges rather than the standard approach which is formulated based on units alone. Such an approach is important as it allows us to understand and subsequently infer any discrepancies between experimental values and theoretical values.

If we equate the new geometric solution (Eqn. 25) with the standard definition (Eqn. 26) we get,

$$R_\infty = \frac{m_e \alpha^2 c}{2h} = \frac{\alpha \phi_e}{8\pi \ell} \qquad \text{Eqn. 27}$$

and thus

$$\alpha = \frac{\phi_e h}{8\pi r_\ell m_e c} = \frac{\phi_e \lambda_e}{8\pi r_\ell} = 7.29735256474256 \times 10^{-3} \qquad \text{Eqn. 28}$$

which is accurate to within 1.65×10^{-12} (99.99999997%) of the CODATA value.

The ratio of the proton mass to the electron mass, μ can also be given in terms of the geometric solution (Eqn. 9 and Eqn. 10),

$$\mu = \frac{m_p}{m_e} = \frac{2\phi m_\ell}{\phi_e m_\ell / 2\alpha} = 4\alpha \frac{\phi}{\phi_e} = 1836.942579077855 \qquad \text{Eqn. 29}$$

Summary

A new derivation for the mass of the electron is presented from first principles, where the mass is defined in terms of the holographic surface-to-volume ratio and the relationship of the electric charge at the Planck scale to that at the electron scale. This new derivation extends the holographic mass solution to the hydrogen Bohr atom and for all known elements, defining the atomic structure and charge as a consequence of the electromagnetic fluctuation of the Planck scale, with an accuracy of 99.99999998%. Furthermore, the atomic number, Z emerges as a natural consequence of this geometric approach. The confinement for both the proton and the electron repulsive electrostatic force are now accounted for by a quantum gravitational force exerted by the granular Planck scale structure of spacetime.

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